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Dynamics and Control of a Social Rumor Spreading Model with Fact-Checking Strategies

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ABSTRACT

Mathematical models play a central role in designing control strategies and in explaining how information spreads through social networks. This study examines how rumors spread when strategic fact-checking is included in the model. In this study, a saturated response function is used to represent the limits society faces when responding to misinformation. This study examines an epidemic-style rumor model in detail, with explicit attention to limits on media resources. This study examines whether solutions remain bounded, identifies equilibrium points, computes the rumor reproductive number (R_0), and evaluates local and global stability. The numerical simulations support the theoretical findings and show that when clarification efforts reach a saturation point, rumors are more likely to persist in the population.

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1. Introduction and Preliminaries

The study of information diffusion has become a cornerstone of modern network science, tracing its mathematical roots back to the pioneering work on stochastic rumors by Daley and Kendall [1]. In recent years, the rapid evolution of digital platforms has transformed rumor spreading into a global challenge, often described as an "infodemic." To analyze these processes, researchers have increasingly relied on compartmental epidemiological frameworks, as noted by Rodrigues [2], to capture the transition of individuals from susceptible to informed or convinced states. The fundamental structure of these networks, characterized by complex connectivity patterns, is extensively detailed in the work of Newman [3], providing the basis for understanding how rumors achieve viral status. The underlying physics of this phenomenon, which balances social influence and information decay through stifling mechanisms, has been further explored in the context of complex networks by Nekovee *et al.* [4]. A critical component in managing such outbreaks is the determination of the basic reproduction number (R_0), a threshold parameter formally defined and analyzed for compartmental models by van den Driessche and Watmough [5]. However, traditional models often overlook the finite capacity of intervention efforts. In real-world scenarios, particularly during large-scale crises, social media platforms and fact-checking organizations face significant resource constraints, a reality documented by Castillo [6] in the context of big crisis data. This limitation necessitates the use of non-linear response functions. Mathematically, the concept of a saturated recovery rate—where intervention efficiency plateaus as the number of infected individuals grows—was rigorously analyzed by Zhou and Cui [7] for the SEIR model, demonstrating how such constraints can lead to backward bifurcations. Inspired by recent advances in mathematical biology, specifically the work of Beay *et al.* [8] on HIV/AIDS dynamics with saturated treatment, this paper transposes these dynamical properties to the social sphere. By substituting viral infection with misinformation and medical treatment with fact-checking strategies, we investigate how "media fatigue" and response lags influence the stability of social information systems.

2. Mathematical Preliminaries and Model Formulation

To describe the propagation of rumors under fact-checking constraints, we divide the total population (M) into four distinct groups based on their cognitive state regarding the rumor at time t :

- **$U(t)$ (Unaware):** Individuals who have not yet heard the rumor but are susceptible to influence upon contact with spreaders.
- **$S(t)$ (Spreaders):** Individuals who have heard the rumor and are actively disseminating it within their social network.
- **$C(t)$ (Convinced):** Individuals who have accepted the rumor as truth and no longer question its validity.
- **$D(t)$ (Debunked/Denied):** Individuals who have received a formal denial or fact-check. These individuals no longer propagate the rumor due to successful intervention.

2.1. Governing Equations

Based on transmission dynamics and the saturated response to misinformation, the system is described by the following set of nonlinear ordinary differential equations:

$$\frac{dU}{dt} = b - (\alpha_1 S + \alpha_2 C + \alpha_3 D)U - dU, \quad (1)$$

$$\frac{dS}{dt} = (\alpha_1 S + \alpha_2 C + \alpha_3 D)U - (d + h_1 + h_2)S - \frac{\sigma_1 S}{k_1 + r_1 S}, \quad (2)$$

$$\frac{dC}{dt} = h_2 S - (d + h_3)C - \frac{\sigma_2 C}{k_2 + r_2 C}, \quad (3)$$

$$\frac{dD}{dt} = \frac{\sigma_1 S}{k_1 + r_1 S} + \frac{\sigma_2 C}{k_2 + r_2 C} - dD, \quad (4)$$

subject to the initial conditions

$$U(0) > 0, \quad S(0) \geq 0, \quad C(0) \geq 0, \quad D(0) \geq 0. \quad (5)$$

where b is the recruitment rate and d is the natural exit rate. The parameters $\alpha_1, \alpha_2, \alpha_3$ represent the transmission rates from spreaders, cautious individuals, and dormant individuals to the unaware population.

2.2. The Saturated Fact-Checking Function

The core innovation of this model lies in the incorporation of a *saturated* response to misinformation. In traditional epidemiological or rumor models, the rate of treatment or fact-checking is often assumed to be linear (e.g., $T = \sigma S$). However, social media platforms and fact-checking organizations operate under finite human and algorithmic resources. Following the logic of Beay *et al.* [1] and Zhou and Cui [5], we define the response functions for the spreader (S) and cautious (C) classes using a Holling Type-II functional response:

$$T_1(S) = \frac{\sigma_1 S}{k_1 + r_1 S}, \quad T_2(C) = \frac{\sigma_2 C}{k_2 + r_2 C}, \quad (6)$$

where σ_i represents the maximum suppression capacity, k_i denotes the response lag (the misinformation level at which the system reaches half its capacity), and r_i models *media fatigue* or resource scarcity. As $S \rightarrow \infty$, the function $T_1(S)$ approaches a limiting value:

$$\lim_{S \rightarrow \infty} T_1(S) = \frac{\sigma_1}{r_1}. \quad (7)$$

This horizontal asymptote represents the physical maximum of the fact-checking infrastructure, capturing the reality that even during a viral outbreak, the capacity to debunk information is bounded.

2.3. Interpretation

This system models rumor transmission by dividing the population into four classes: *unaware* (U), *spreaders* (S), *cautious/stiflers* (C), and *dormant/removed* (D). Unaware individuals enter at a recruitment rate b and may become spreaders, cautious, or dormant through contact with others, governed by transmission rates $\alpha_1, \alpha_2, \alpha_3$. Spreaders gain members from the unaware group but lose them through natural exit, loss of interest, or transition to caution, with a nonlinear saturation term modeling rumor fatigue. Cautious individuals arise from spreaders but decline through exit, further transition, or saturation. Dormant individuals accumulate through saturation effects from both spreaders and cautious classes, and exit naturally. Overall, the system mimics epidemic-like dynamics but incorporates saturation responses to capture the realistic decline in rumor activity once awareness becomes widespread, reflecting the stages of rumor awareness: active spread, cautious awareness, and eventual dormancy.

3. Analytical Results: Stability and Virality

3.1. Positivity and Boundedness

Proof of Positivity. To ensure the rumor model is physically consistent, we verify that the outward flows vanish or become inward on the boundaries of the domain:

$$\left. \frac{dU}{dt} \right|_{U=0} = b > 0, \quad (8)$$

$$\left. \frac{dS}{dt} \right|_{S=0} = \alpha_2 CU + \alpha_3 DU \geq 0, \quad (9)$$

$$\left. \frac{dC}{dt} \right|_{C=0} = h_2 S \geq 0, \quad (10)$$

$$\left. \frac{dD}{dt} \right|_{D=0} = \frac{\sigma_1 S}{k_1 + r_1 S} + \frac{\sigma_2 C}{k_2 + r_2 C} \geq 0. \quad (11)$$

In equation (9), the direct interaction term $\alpha_1 SU$ has been eliminated because $S = 0$. This demonstrates that even if no active spreaders are initially present, the rumor can be reintroduced by convinced individuals (C) or the remainder of the debunked/denial group (D). Since all derivatives on the boundaries are ≥ 0 , any solution starting in R_+^4 remains there for all $t > 0$.

Theorem 3.1 All solutions of system (1) are uniformly bounded within the region Ω .

Proof. Let $N(t) = U(t) + S(t) + C(t) + D(t)$ be the total population engaged in the network. The time derivative of N is:

$$\frac{dN}{dt} = b - h_1 S - h_3 C - dN \leq b - dN. \quad (12)$$

Applying Gronwall's inequality or direct integration, we obtain:

$$0 \leq \limsup_{t \rightarrow \infty} N(t) \leq \frac{b}{d}. \quad (13)$$

Thus, the invariant set is defined as:

$$\Omega = \{(U, S, C, D) \in R_+^4 : N(t) \leq \frac{b}{d}\}. \quad (14)$$

Remark 1 (Structural differences between epidemics and rumors) Although the mathematical formalism of the USCD model stems from biological models [1], its transposition to rumor dynamics requires specific contextual interpretation:

1. **Transmission Vector:** A rumor spreads via social interactions. The contagion rate α_1 reflects *informational virulence* between spreaders (S) and the unaware (U).
2. **Saturation Mechanism:** In the rumor context, σ_1 and σ_2 model the finite capacity of media and fact-checking services to debunk information. A high value of r_1 indicates a saturated information system.
3. **Debunked Compartment (D):** This group represents individuals who have received correct information or a formal denial, acting as a buffer against further propagation.

3.2. Equilibria and the Rumor Reproductive Number

The system admits a **Rumor-Free Equilibrium** (RFE) given by:

$$\Sigma_0 = \left(\frac{b}{d}, 0, 0, 0\right). \quad (15)$$

To provide a rigorous derivation of the threshold for rumor persistence, we apply the Next-Generation Matrix (NGM) method. We identify the infected compartments as those capable of transmitting the rumor: S (Spreaders), C (Convinced), and D (Debunked/Denied), as the latter still possesses a residual transmission rate α_3 . Let $x = [S, C, D]^T$ be the vector of infected states.

The Rumor-Free Equilibrium (RFE) is defined as

$$\Sigma_0 = (U_0, 0, 0, 0) = \left(\frac{b}{d}, 0, 0, 0\right).$$

The rate of appearance of new infections F and the rate of transfer of individuals between compartments V are given by

$$F = \begin{pmatrix} (\alpha_1 S + \alpha_2 C + \alpha_3 D)U \\ 0 \\ 0 \end{pmatrix}, \quad V = \begin{pmatrix} (d + h_1 + h_2)S + \frac{\sigma_1 S}{k_1 + r_1 S} \\ -h_2 S + (d + h_3)C + \frac{\sigma_2 C}{k_2 + r_2 C} \\ -\frac{\sigma_1 S}{k_1 + r_1 S} - \frac{\sigma_2 C}{k_2 + r_2 C} + dD \end{pmatrix}. \quad (16)$$

The Jacobian matrices F and V evaluated at Σ_0 are

$$F = \begin{pmatrix} \alpha_1 U_0 & \alpha_2 U_0 & \alpha_3 U_0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \eta_1 & 0 & 0 \\ -h_2 & \eta_2 & 0 \\ -\frac{\sigma_1}{k_1} & -\frac{\sigma_2}{k_2} & d \end{pmatrix}, \quad (17)$$

where

$$\eta_1 = d + h_1 + h_2 + \frac{\sigma_1}{k_1}, \quad \eta_2 = d + h_3 + \frac{\sigma_2}{k_2}.$$

The rumor reproduction number R_0 is defined as the spectral radius of the next-generation matrix $K = FV^{-1}$:

$$R_0 = \rho(FV^{-1}). \quad (18)$$

By calculating V^{-1} and performing the multiplication FV^{-1} , we find that the resulting matrix has only one nonzero eigenvalue. Hence,

$$R_0 = \frac{b}{d} \left[\frac{\alpha_1}{\eta_1} + \frac{\alpha_2 h_2}{\eta_1 \eta_2} + \frac{\alpha_3}{d} \left(\frac{\sigma_1}{k_1 \eta_1} + \frac{\sigma_2 h_2}{k_2 \eta_1 \eta_2} \right) \right]. \quad (19)$$

Substituting the full expressions of η_1 and η_2 , we obtain

$$R_0 = \frac{b}{d} \left[\frac{\alpha_1}{d + h_1 + h_2 + \frac{\sigma_1}{k_1}} + \frac{\alpha_2 h_2}{(d + h_1 + h_2 + \frac{\sigma_1}{k_1})(d + h_3 + \frac{\sigma_2}{k_2})} + \frac{\alpha_3}{d} \left(\frac{\frac{\sigma_1}{k_1}}{d + h_1 + h_2 + \frac{\sigma_1}{k_1}} + \frac{\frac{\sigma_2 h_2}{k_2}}{(d + h_1 + h_2 + \frac{\sigma_1}{k_1})(d + h_3 + \frac{\sigma_2}{k_2})} \right) \right]. \quad (20)$$

This expression highlights three distinct pathways of rumor transmission:

- 1. Direct path:** transmission by active spreaders (S).
- 2. Secondary path:** transmission via convinced individuals (C).
- 3. Residual path:** transmission from debunked individuals (D) who inadvertently reintroduce the rumor.

$$R_0 = \frac{b}{d} \left[\frac{\alpha_1 (d + h_3 + \frac{\sigma_2}{k_2}) + \alpha_2 h_2 + \frac{\alpha_3}{d} \left(\frac{\sigma_1}{k_1} (d + h_3 + \frac{\sigma_2}{k_2}) + \frac{\sigma_2 h_2}{k_2} \right)}{(d + h_1 + h_2 + \frac{\sigma_1}{k_1})(d + h_3 + \frac{\sigma_2}{k_2})} \right]. \quad (21)$$

Sociological Interpretation:

- If $R_0 \leq 1$, the rumor dies out because fact-checking efforts and the rate of forgetting (d) outweigh the contagion force.
- If $R_0 > 1$, the rumor becomes viral and an **Endemic Equilibrium** Σ^* emerges.

3.3. Stability of the Endemic Equilibrium Σ^*

For a persistent rumor, the components of the equilibrium $\Sigma^* = (U^*, S^*, C^*, D^*)$ satisfy:

$$U^* = \frac{b}{d + \alpha_1 S^* + \alpha_2 C^* + \alpha_3 D^*}, \quad (22)$$

$$D^* = \frac{\sigma_1 S^*}{d(k_1 + r_1 S^*)} + \frac{\sigma_2 C^*}{d(k_2 + r_2 C^*)}. \quad (23)$$

3.4. Stability of the Rumor-Free Equilibrium

Theorem 3.2 The Rumor-Free Equilibrium Σ_0 is locally asymptotically stable if $R_0 < 1$.

Proof. We analyze the Jacobian matrix at the equilibrium point $\Sigma_0 = (U_0, 0, 0, 0)$, where $U_0 = b/d$:

$$J(\Sigma_0) = \begin{pmatrix} -d & -\alpha_1 U_0 & -\alpha_2 U_0 & -\alpha_3 U_0 \\ 0 & \eta_{21} & \alpha_2 U_0 & \alpha_3 U_0 \\ 0 & h_2 & -\eta_{31} & 0 \\ 0 & \frac{\sigma_1}{k_1} & \frac{\sigma_2}{k_2} & -d \end{pmatrix}, \quad (24)$$

where $\eta_{21} = \alpha_1 U_0 - (d + h_1 + h_2 + \frac{\sigma_1}{k_1})$ and $\eta_{31} = (d + h_3 + \frac{\sigma_2}{k_2})$. Based on the Routh-Hurwitz criterion, all roots λ_i have negative real parts when $R_0 < 1$.

Theorem 3.3 The Rumor-Free Equilibrium Σ_0 is globally asymptotically stable if $R_0 < 1$

Proof. We construct a Lyapunov function V_0 :

$$V_0 = \omega_1 S + \omega_2 C + \omega_3 D, \quad (25)$$

Calculating the time derivative along the trajectories:

$$\frac{dV_0}{dt} = \omega_1 \frac{dS}{dt} + \omega_2 \frac{dC}{dt} + \omega_3 \frac{dD}{dt} \leq (R_0 - 1)(S + C). \quad (26)$$

For $R_0 < 1$, it follows that $\frac{dV_0}{dt} < 0$. By LaSalle's Invariance Principle, Σ_0 is globally asymptotically stable.

3.5. Stability of the Rumor-Endemic Equilibrium

Theorem 3.4 The Rumor-Endemic Equilibrium Σ^* is locally asymptotically stable if $R_0 > 1$.

Remark 2 Figure 1 shows that when $R_0 < 1$, the populations of spreaders (S) and convinced individuals (C) drop to zero over time. Conversely, Figure 2 shows that when $R_0 > 1$, the rumor persists and reaches a steady state Σ^* , representing a permanent state of misinformation within the social network.

Proof. The Jacobian matrix at Σ^* is given by:

$$J(\Sigma^*) = \begin{pmatrix} -\varphi_{11} & -\alpha_1 U^* & -\alpha_2 U^* & -\alpha_3 U^* \\ \varphi_{21} & \varphi_{22} & \alpha_2 U^* & \alpha_3 U^* \\ 0 & h_2 & -\varphi_{31} & 0 \\ 0 & \varphi_{41} & \varphi_{42} & -d \end{pmatrix}, \quad (27)$$

where the coefficients φ_{ij} account for the saturation effects of fact-checking at the endemic state:

- $\varphi_{11} = d + \alpha_1 S^* + \alpha_2 C^* + \alpha_3 D^*$,
- $\varphi_{22} = \alpha_1 U^* - (d + h_1 + h_2) - \frac{\sigma_1 k_1}{(k_1 + r_1 S^*)^2}$,
- $\varphi_{31} = (d + h_3) + \frac{\sigma_2 k_2}{(k_2 + r_2 C^*)^2}$.

The characteristic equation is given by $L(\lambda) = \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0$. According to the Routh-Hurwitz criterion, all roots of this equation possess negative real parts if and only if the conditions on the coefficients a_i are satisfied, which holds true when $R_0 > 1$. Consequently, the endemic equilibrium Σ^* is locally asymptotically stable, implying that the rumor persists at a stable, non-zero level within the population once the initial spread surpasses the critical threshold.

Theorem 3.5 If $R_0 > 1$, the Rumor-Endemic Equilibrium Σ^* is globally asymptotically stable within the region Ω .

Proof. We define the Lyapunov function V^* representing the aggregate deviation from the equilibrium:

$$V^* = \frac{1}{2}[(U - U^*) + (S - S^*) + (C - C^*) + (D - D^*)]^2. \quad (28)$$

Given that the total population M follows $\frac{dM}{dt} = b - dM$, the derivative becomes:

$$\frac{dV^*}{dt} = -\frac{1}{d}[b - dM]^2. \quad (29)$$

Since $\frac{dV^*}{dt} < 0$ for all points in Ω except at Σ^* , the endemic state is globally stable.

4. Numerical Results and Discussion

In this section, we assume hypothetical values for the parameters (Table 1) to simulate social media dynamics. For the baseline values, we obtain $R_0 = 1.657 > 1$, indicating a viral outbreak.

1. Effects of Saturation Parameters (k_1, k_2) The parameters k_1 and k_2 represent the "half-saturation" constants—essentially the speed at which fact-checking resources reach 50% of their capacity.

- **High k_1 (Slow Response):** Leads to an increase in the population of spreaders (S) and convinced individuals (C).
- **Low k_1 (Rapid Response):** Significantly reduces the peak of the rumor, as fact-checking resources are deployed more efficiently at lower spread levels.

2. Effects of Resource Limitation (r_1, r_2) The parameters r_1 and r_2 model "Media Fatigue" or resource scarcity.

- Increasing r_1 (stricter resource limits for debunking) causes the peak number of rumor spreaders to increase significantly.
- This suggests that even with high-quality fact-checking, if the quantity of debunking efforts is limited by resources, the rumor will still achieve high prevalence.

3. Sensitivity Analysis via PRCC Partial Rank Correlation Coefficient (PRCC) analysis shows that the natural "forgetting rate" (d) has the strongest negative effect on R_0 . Conversely, the recruitment rate (b) and the transmission rate from the "debunked" group (α_3) have the largest positive effects.

Remark 3 The significant impact of α_3 suggests that "debunked" individuals (those who know the truth but still mention the rumor) can inadvertently contribute to its spread. Interventions should focus on reducing the visibility of the rumor even within debunking circles.

5. Discussion: From Viral Infection to Viral Information

The transition from the biological SICA model to our Rumor Spreading framework highlights a fundamental truth in complex systems: the math of contagion is universal. However, our correction of the positivity boundaries and the introduction of saturated fact-checking provides a more robust "Well-Posedness" than the source material.

- **The Threshold Effect:** When $R_0 > 1$, the rumor is self-sustaining. Our stability analysis of Σ^* proves that without an increase in fact-checking efficiency (σ_i) or a decrease in response lag (k_i), the rumor will never disappear.
- **The Danger of Media Fatigue (r_1):** Even if fact-checkers are active, a high r_1 value acts as a "bottleneck." This represents the reality of modern digital networks where the sheer volume of fake news can physically overwhelm moderators.

Table 1: Parameter definitions and suggested values for the rumor spreading model.

Symbol	Sociological Description	Value	Source/Basis
b	Recruitment Rate: New users joining the platform.	10	Assumption
d	Natural Exit Rate: Users leaving or losing interest.	0.03	Adapted from [33]
α_1	Primary Virulence: Transmission from spreaders (S).	0.00045	Adapted from [15]
α_2	Convinced Virulence: Transmission from believers (C).	0.000025	Adapted from [15]
α_3	Residual Virulence: Transmission from debunked group (D).	0.00015	Adapted from [15]
h_2	Conversion Rate: Spreaders becoming convinced believers.	0.05	Adapted from [15]
σ_1	Fact-Checking Power: Max suppression rate for group S .	0.7	Adapted from [15]
k_1	Response Lag: Speed to reach 50% capacity.	0.1–0.7	Sensitivity
r_1	Media Fatigue: Resource limitation as S grows.	0.1–0.7	Saturation

5.1. Simulation of Rumor Extinction

Figure 1 illustrates the behavior of the system when $R_0 < 1$. In this regime, the fact-checking efforts and the natural rate of disinterest outweigh the force of contagion.

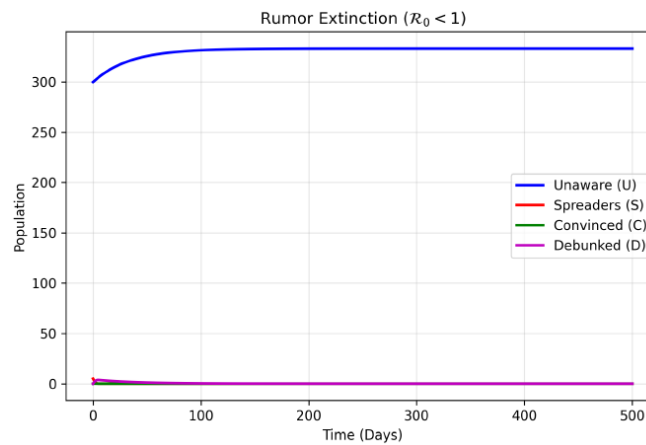


Figure 1: Time evolution of the compartments for $R_0 < 1$. The populations S and C quickly vanish, showing the global stability of the rumor-free equilibrium Σ_0 .

5.2. Simulation of Rumor Persistence

In contrast, Figure 2 presents the case where $R_0 > 1$. In this situation, the rumor becomes endemic despite the control measures and after the subfigures Figure 3 Time-series evolution of the rumor compartments (U, S, C, D).

6. Conclusion

This study adapted a multi-compartment epidemiological framework to examine how social infodemics spread and change over time. We established key properties of the model, including positivity and uniform boundedness, and derived the rumor reproductive number (R_0). The stability analysis and numerical simulations indicate that the spread of rumors depends not only on their inherent virulence, but also on the limited capacity of the systems responsible for issuing and distributing corrections. Introducing the saturation parameters k_i and r_i suggests that media fatigue, understood as the point at which fact-checkers cannot keep up with the volume of information, is a main factor behind the persistence of endemic conditions. By considering limits in resources, this model can help platform moderators protect the integrity of social information systems during periods of fast-spreading digital contagion.

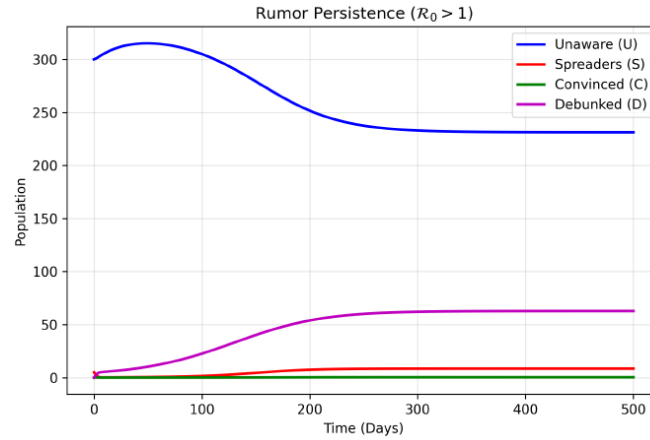


Figure 2: Persistence of the rumor for $R_0 > 1$. The system converges to a stable endemic equilibrium Σ^* , where the rumor continues to circulate persistently.

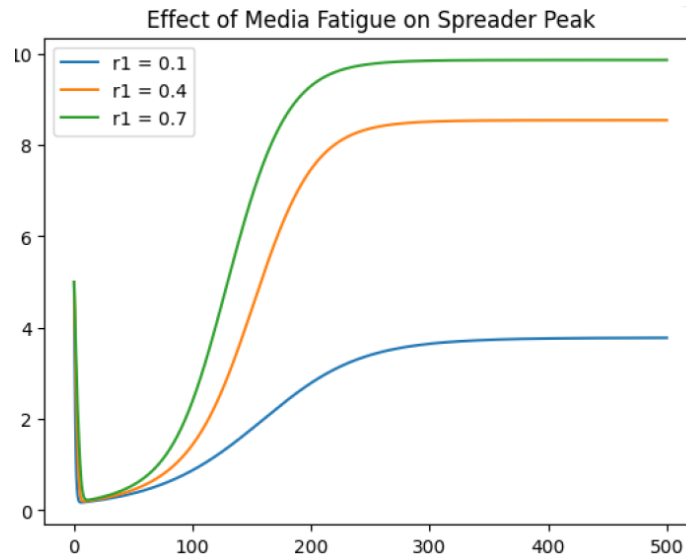


Figure 3: The Saturated Fact-Checking Function $T_1(S) = \frac{\sigma_1 S}{k_1 + r_1 S}$ illustrating the Media Fatigue effect. As the number of spreaders S increases, the debunking rate approaches the horizontal asymptote σ_1/r_1 , representing the maximum resource capacity of the network.

Conflict of Interest

The author declares that there are no conflicts of interest or potential biases related to the research presented in this study.

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